DPP - Daily Practice Problems

Name : Date :

Start Time : End Time :

PHYSICS 14

SYLLABUS : Centre of mass and its motion

Max. Marks : 112 **Time : 60 min.**

GENERAL INSTRUCTIONS

- The Daily Practice Problem Sheet contains 28 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.
- You have to evaluate your Response Grids yourself with the help of solution booklet.
- Each correct answer will get you 4 marks and 1 mark shall be deduced for each incorrect answer. No mark will be given/ deducted if no bubble is filled. Keep a timer in front of you and stop immediately at the end of 60 min.
- The sheet follows a particular syllabus. Do not attempt the sheet before you have completed your preparation for that syllabus. Refer syllabus sheet in the starting of the book for the syllabus of all the DPP sheets.
- After completing the sheet check your answers with the solution booklet and complete the Result Grid. Finally spend time to analyse your performance and revise the areas which emerge out as weak in your evaluation.

DIRECTIONS (Q.1-Q.20) : There are 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** choice is correct.

Q.1 In the *HCl* molecule, the separation between the nuclei of the two atom is about 1.27 A° (1A° = 10^{-10} m). The approximate location of the centre of mass from the hydrogen atom, assuming the chlorine atom to be about 35.5 times massive as hydrogen is

(a) 1 Å (b) 2.5 Å (c) 1.24 Å (d) 1.5 Å

Q.2 A 2 kg body and a 3 kg body are moving along the x-axis. At a particular instant the 2 kg body has a velocity of 3 ms^{-1} and the 3 kg body has the velocity of 2ms^{-1}. The velocity of the centre of mass at that instant is

(a) 5 ms^{-1} (b) 1 ms^{-1}
(c) 0 (d) None of these

Q.3 The distance between the carbon atom and the oxygen atom in a carbon monoxide molecule is 1.1 Å. Given, mass of carbon atom is 12 a.m.u. and mass of oxygen atom is 16 a.m.u., calculate the position of the centre of mass of the carbon monoxide molecule

(a) 6.3 Å from the carbon atom
(b) 1 Å from the oxygen atom
(c) 0.63 Å from the carbon atom
(d) 0.12 Å from the oxygen atom

Q.4 The velocities of three particles of masses 20g, 30g and 50g are $10\hat{i}, 10\hat{j}$ and $10\hat{k}$ respectively. The velocity of the centre of mass of the three particles is

(a) $2\hat{i}+3\hat{j}+5\hat{k}$ (b) $10(\hat{i}+\hat{j}+\hat{k})$
(c) $20\hat{i}+30\hat{j}+5\hat{k}$ (d) $2\hat{i}+30\hat{j}+50\hat{k}$

RESPONSE GRID 1. ⓐⓑⓒⓓ 2. ⓐⓑⓒⓓ 3. ⓐⓑⓒⓓ 4. ⓐⓑⓒⓓ

Space for Rough Work

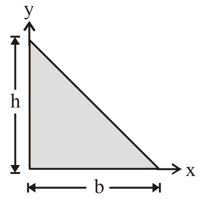
Q.5 The centre of mass of a triangle shown in figure has coordinates

(a) $x=\frac{h}{2},\ y=\frac{b}{2}$

(b) $x=\frac{b}{2},\ y=\frac{h}{2}$

(c) $x=\frac{b}{3},\ y=\frac{h}{3}$

(d) $x=\frac{h}{3},\ y=\frac{b}{3}$



Q.6 Two bodies of masses 2 kg and 4 kg are moving with velocities 2 m/s and 10 m/s respectively along same direction. Then the velocity of their centre of mass will be

(a) 8.1 m/s (b) 7.3 m/s
(c) 6.4 m/s (d) 5.3 m/s

Q.7 Four particles of masses m, $2m$, $3m$ and $4m$ are arranged at the corners of a parallelogram with each side equal to a and one of the angle between two adjacent sides is 60°. The parallelogram lies in the x-y plane with mass m at the origin and 4m on the x-axis. The centre of mass of the arrangement will be located at

(a) $\left(\frac{\sqrt{3}}{2}a, 0.95a\right)$ (b) $\left(0.95a, \frac{\sqrt{3}}{4}a\right)$

(c) $\left(\frac{3a}{4}, \frac{a}{2}\right)$ (d) $\left(\frac{a}{2}, \frac{3a}{4}\right)$

Q.8 Three identical metal balls each of radius r are placed touching each other on a horizontal surface such that an equilateral triangle is formed, when centres of three balls are joined. The centre of the mass of system is located at

(a) Horizontal surface
(b) Centre of one of the balls
(c) Line joining centres of any two balls
(d) Point of intersection of the medians

Q.9 2 bodies of different masses of 2 kg and 4 kg are moving with velocities 20 m/s and 10 m/s towards each other due to mutual gravitational attraction. What is the velocity of their centre of mass?

(a) 5 m/s (b) 6 m/s
(c) 8 m/s (d) Zero

Q.10 Two particles of masses m_1 and m_2 initially at rest start moving towards each other under their mutual force of attraction. The speed of the centre of mass at any time t, when they are at a distance r apart, is

(a) zero (b) $\left(G\frac{m_1m_2}{r^2}.\frac{1}{m_1}\right)t$

(c) $\left(G\frac{m_1m_2}{r^2}.\frac{1}{m_2}\right)t$ (d) $\left(G\frac{m_1m_2}{r^2}.\frac{1}{m_1+m_2}\right)t$

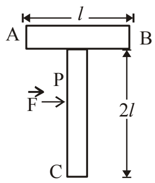
Q.11 A 'T' shaped object, dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C

(a) $\frac{4}{3}l$

(b) l

(c) $\frac{2}{3}l$

(d) $\frac{3}{2}l$



Q.12 Two spheres of masses $2M$ and M are initially at rest at a distance R apart. Due to mutual force of attraction, they approach each other. When they are at separation $R/2$, the acceleration of the centre of mass of spheres would be

(a) 0 m/s^2 (b) g m/s^2
(c) 3 g m/s^2 (d) 12 g m/s^2

Q.13 Masses 8 kg, 2 kg, 4 kg and 2 kg are placed at the corners A, B, C, D respectively of a square ABCD of diagonal 80 cm. The distance of centre of mass from A will be

(a) 20 cm (b) 30 cm
(c) 40 cm (d) 60 cm

Q.14 If linear density of a rod of length 3m varies as $\lambda = 2 + x$, them the position of the centre of gravity of the rod is

(a) $\frac{7}{3}$m (b) $\frac{12}{7}$m

(c) $\frac{10}{7}$m (d) $\frac{9}{7}$m

RESPONSE GRID					
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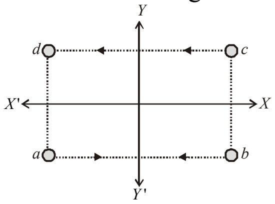
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

Space for Rough Work

Q.15 Four bodies of equal mass start moving with same speed as shown in the figure. In which of the following combination the centre of mass will remain at origin?



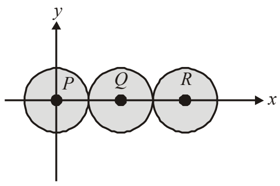
(a) c and d

(b) a and b

(c) a and c

(d) b and d

Q.16 Three identical spheres, each of mass 1 kg are kept as shown in figure, touching each other, with their centres on a straight line. If their centres are marked P, Q, R respectively, the distance of centre of mass of the system from P is



(a) $\frac{PQ+PR+QR}{3}$

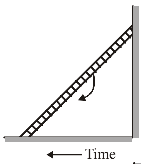
(b) $\frac{PQ+PR}{3}$

(c) $\frac{PQ+QR}{3}$

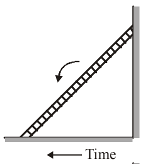
(d) $\frac{PR+QR}{3}$

Q.17 A ladder is leaned against a smooth wall and it is allowed to slip on a frictionless floor. Which figure represents trace of motion of its centre of mass

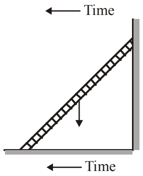
(a)



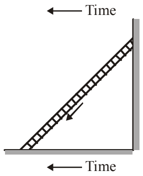
(b)



(c)



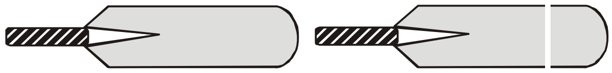
(d)



Q.18 The two particles X and Y, initially at rest, start moving towards each other under mutual attraction. If at any instant the velocity of X is V and that of Y is $2V$, the velocity of their centre of mass will be

(a) 0 (b) V (c) $2V$ (d) $V/2$

Q.19 A cricket bat is cut at the location of its centre of mass as shown in the fig. Then



(a) The two pieces will have the same mass

(b) The bottom piece will have larger mass

(c) The handle piece will have larger mass

(d) Mass of handle piece is double the mass of bottom piece

Q.20 Consider a system of two particles having mass m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass of particles through a distance d, by what distance would be particle of mass m_2 move so as to keep the centre of mass of particles at the original position?

(a) $\frac{m_1}{m_1+m_2}d$ (b) $\frac{m_1}{m_2}d$ (c) d (d) $\frac{m_2}{m_1}d$

DIRECTIONS (Q.21-Q.22) : In the following questions, more than one of the answers given are correct. Select the correct answers and mark it according to the following codes:

Codes :

(a) 1, 2 and 3 are correct **(b)** 1 and 2 are correct

(c) 2 and 4 are correct **(d)** 1 and 3 are correct

Q.21 Choose the wrong statements about the centre of mass (CM) of a system of two particles

(1) The CM lies on the line joining the two particles midway between them

(2) The CM lies on the line joining them at a point whose distance from each particle is proportional to the square of the mass of that particle

(3) The CM is on the line joining them at a point whose distance from each particle is proportional to the mass of that particle

(4) The CM lies on the line joining them at a point whose distance from each particle is inversely proportional to the mass of that particle

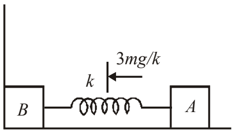
RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ			

Space for Rough Work

Q.22 Choose the wrong statements about the centre of mass of a body
(1) It lies always outside the body
(2) It lies always inside the body
(3) It lies always on the surface of the body
(4) It may lie within, outside or on the surface of the body

DIRECTIONS (Q.23-Q.25) : Read the passage given below and answer the questions that follows :

A system consists of block A and B each of mass m connected by a light spring as shown in the figure with block B in contact with a wall. The block A compresses the spring by $3mg/k$ from natural length of spring and then released from rest. Neglect friction anywhere.



Q.23 Acceleration of centre of mass of system comprising A and B just after A is released is
(a) 0 (b) $3g/2$
(c) $3g$ (d) None of these

Q.24 Velocity of centre of mass of system comprising A and B when block B just loses contact with the wall
(a) $3g\sqrt{\frac{m}{k}}$ (b) $\frac{3g}{2}\sqrt{\frac{m}{k}}$
(c) $2g\sqrt{\frac{m}{k}}$ (d) None of these

Q.25 Maximum extension in the spring after system loses contact with wall
(a) $\frac{3mg}{\sqrt{2}k}$ (b) $\frac{\sqrt{3}mg}{2k}$
(c) $\frac{\sqrt{3}mg}{\sqrt{2}k}$ (d) None of these

DIRECTIONS (Q. 26-Q.28) : Each of these questions contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement -1 is False, Statement-2 is True.
(d) Statement -1 is True, Statement-2 is False.

Q.26 **Statement-1 :** The centre of mass of a system of n particles is the weighted average of the position vector of the n particles making up the system.
Statement-2 : The position of the centre of mass of a system is independent of coordinate system.

Q.27 **Statement-1 :** The centre of mass of a proton and an electron, released from their respective positions remains at rest.
Statement-2 : The centre of mass remains at rest, if no external force is applied.

Q.28 **Statement-1 :** Position of centre of mass is independent of the reference frame.
Statement-2 : Centre of mass is same for all bodies.

RESPONSE GRID					
	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ			

DAILY PRACTICE PROBLEM SHEET 14 - PHYSICS			
Total Questions	28	Total Marks	112
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	28	Qualifying Score	46
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work

DAILY PRACTICE PROBLEMS

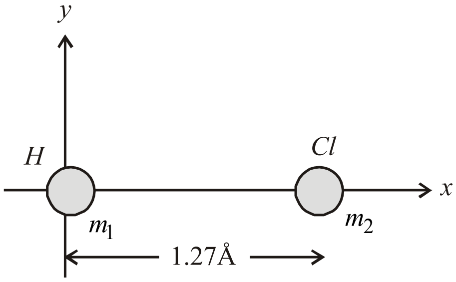
PHYSICS SOLUTIONS

14

1. (c) $m_1 = 1, m_2 = 35.5_1, \vec{r_1} = 0, \vec{r_2} = 1.27\hat{i}$

$$\vec{r} = \frac{m_1\vec{r_1} \times m_2\vec{r_2}}{m_1 + m_2} \Rightarrow \vec{r} = \frac{35.5 \times 1.27}{1 + 35.5}\hat{i}$$

$$\vec{r} = \frac{35.5}{36.5} \times 1.27\hat{i} = 1.24\hat{i}$$



2. (d) $\vec{v}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1 + m_2}$

$$= \frac{2 \times 3 + 3 \times 2}{2 + 3} = \frac{12}{5} = 2.4 \text{ m/s}$$

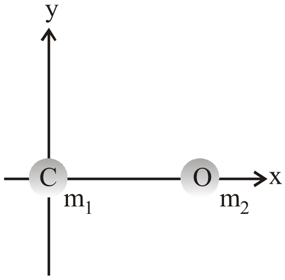
3. (c) $m_1 = 12, m_2 = 16$

$\vec{r_1} = 0\hat{i} + 0\hat{j}, r_2 = 1.1\hat{i} + 0\hat{j}$

$$\vec{r_1} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2}$$

$$\vec{r_1} = \frac{16 \times 1.1}{28}\hat{i} = 0.63\hat{i}$$

i.e. 0.63Å from carbon atom.



4. (a) $\vec{v}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3}}{m_1 + m_2 + m_3}$

$$= \frac{20 \times 10\hat{i} + 30 \times 10\hat{j} + 50 \times 10\hat{k}}{100}$$

$$\therefore v_{cm} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

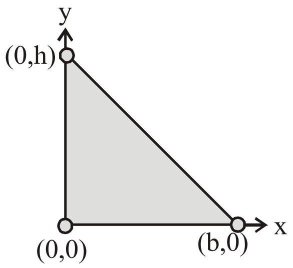
5. (c) We can assume that three particles of equal mass m are placed at the corners of triangle.

$\vec{r_1} = 0\hat{i} + 0\hat{j}, \vec{r_2} = b\hat{i} + 0\hat{j}$

and $\vec{r_3} = 0\hat{i} + h\hat{j}$

$$\therefore \overline{r_{cm}} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3}}{m_1 + m_2 + m_3}$$

$$= \frac{b}{3}\hat{i} + \frac{h}{3}\hat{j}$$

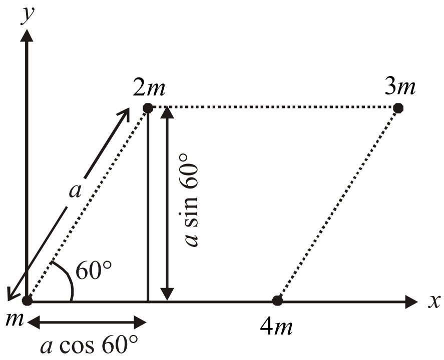


i.e. coordinates of centre of mass is $\left(\frac{b}{3}, \frac{h}{3}\right)$

6. (b) $\vec{v}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1 + m_2}$

$$= \frac{2 \times 2 + 4 \times 10}{2 + 4} = 7.3 \text{ m/s}$$

7. (b) Let $m_1 = m, m_2 = 2m, m_3 = 3m, m_4 = 4m$



$$\vec{r_1} = 0\hat{i} + 0\hat{j}$$

$$\vec{r_2} = a\cos 60\hat{i} + a\sin 60\hat{j} = \frac{a}{2}i + \frac{a\sqrt{3}}{2}\hat{j}$$

$$\vec{r_3} = (a + a\cos 60)\hat{i} + a\sin 60\hat{j} = \frac{3}{2}a\hat{i} + \frac{a\sqrt{3}}{2}\hat{j}$$

$$\vec{r_4} = a\hat{i} + 0\hat{j}$$

by substituting above value in the following formula

$$\vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + m_4\vec{r_4}}{m_1 + m_2 + m_3 + m_4} = 0.95a\hat{i} + \frac{\sqrt{3}}{4}a\hat{j}$$

So the location of centre of mass $\left[0.95a, \frac{\sqrt{3}}{4}a\right]$

8. (d)

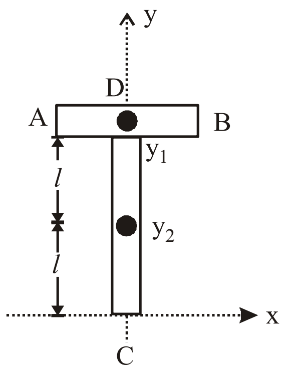
9. (d) $m_1 = 2\text{kg}, m_2 = 4\text{kg}, \vec{v_1} = 2m/s, \vec{v_2} = -10m/s$

$$\vec{v}_{cm} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1 + m_2}$$

$$= \frac{2 \times 20 - 4 \times 10}{2 + 4} = 0m/s$$

10. (a) As initially both the particles were at rest therefore velocity of centre of mass was zero and there is no external force on the system so speed of centre of mass remains constant i.e. it should be equal to zero.

11. (a) For translatory motion the force should be applied on the centre of mass of the body, so we have to calculate the location of centre of mass of 'T' shaped object.
Let mass of rod AB is *m* so the mass of rod CD will be 2m.
Let y_1 is the centre of mass of rod AB and y_2 is the centre of mass of rod CD. We can consider that whole mass of the rod is placed at their respective centre of mass i.e., mass m is placed at y_1 and mass 2 m is placed at y_2.



Taking point 'C' at the origin, position vector of point y_1 and y_2 can be written as $\vec{r}_1 = 2l\,\hat{j}, \vec{r}_2 = l\,\hat{j}$, and $m_1 = m$ and $m_2 = 2m$
Position vector of centre of mass of the system

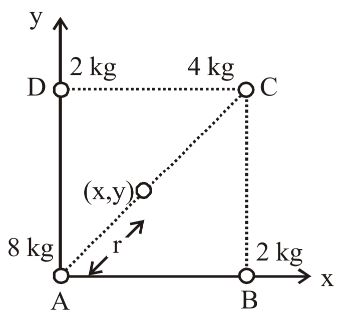
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{m2l\,\hat{j} + 2ml\,\hat{j}}{m + 2m}$$

$$= \frac{4ml\,\hat{j}}{3m} = \frac{4}{3}l\,\hat{j}$$

Hence the distance of centre of mass from $C = \frac{4}{3}l$

12. **(a)** Initial acceleration is zero of the system. So it will always remain zero because there is no external force on the system.

13. **(b)** According to figure let A is the origin and co-ordinates of centre of mass be (x, y) then,



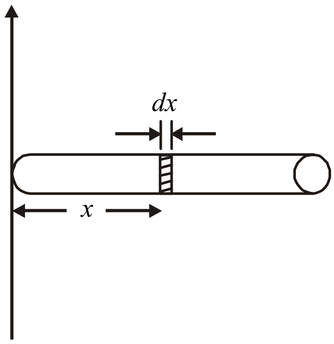
$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{0 + 2\times\frac{80}{\sqrt{2}} + 4\times\frac{80}{\sqrt{2}} + 0}{16} = \frac{30}{\sqrt{2}}$$

Similarly $y = \frac{30}{\sqrt{2}}$ so, $r = \sqrt{x^2 + y^2} = 30\,\text{cm}$

14. (b) Linear density of the rod varies with distance

$$\frac{dm}{dx} = \lambda\ (Given)\ \therefore\ dm = \lambda dx$$



Position of centre of mass

$$x_{cm} = \frac{\int dm \times x}{\int dm}$$

$$= \frac{\int_0^3 (\lambda\, dx)\times x}{\int_0^3 \lambda\, dx}$$

$$= \frac{\int_0^3 (2+x)\times x dx}{\int_0^3 (2+x)dx} = \frac{\left[x^2 + \frac{x^3}{3}\right]_0^3}{\left[2x + \frac{x^3}{2}\right]_0^3}$$

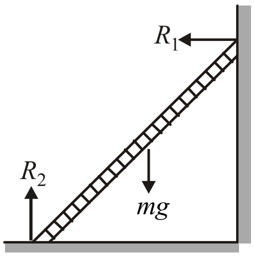
$$= \frac{9+9}{6+\frac{9}{2}} = \frac{36}{21} = \frac{12}{7}m.$$

15. (c) Centre of mass lies always on the line that joins the two particles.
For the combination *cd* and *ab* this line does not pass through the origin.
For combination *bd*, initially it pass through the origin but later on it moves toward negative *x*-axis.
But for combination *ac* it will always pass through origin. So we can say that centre of mass of this combination will remain at origin.

16. (b) $x_{cm} = \frac{1\times 0+1\times PQ+1\times PR}{1+1+1} = \frac{PQ+PR}{3}$

and $y_{cm} = 0$

17. (a)



Due to net force in downward direction and towards left centre of mass will follow the path as shown in figure.

18. (a) Initially both the particles were at rest so $v_{cm} = 0$. As external force on the system is zero therefore velocity of centre of mass remains unaffected.

19. (a) $m_1\vec{r}_1 + m_2\vec{r}_2 = 0$

$\Rightarrow \frac{m}{4}15\hat{j} + \frac{3m}{4}\vec{r}_2 = 0$

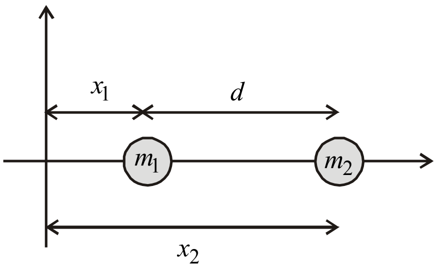
$\Rightarrow \vec{r}_2 = -5\hat{j}$

i.e. larger fragment is at $y = -5$ cm.

20. (b) Centre of mass is closer to massive part of the body therefore the bottom piece of bat have larger mass.

21. (b) Initial position of centre of mass

$$r_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} \quad \text{...(i)}$$



If the particle of mass m_1 is pushed towards the centre of mass of the system through distance d and to keep the centre of mass at the original position let second particle displaced through distance d' away from the centre of mass.

Now $r_{cm} = \frac{m_1(x_1+d)+m_2(x_2+d')}{m_1+m_2}$ ...(ii)

Equating (i) and (ii)

$$\frac{m_1x_1 + +m_2x_2}{m_1+m_2} = \frac{m_1(x_1+d)+m_2(x_2+d')}{m_1+m_2}$$

By solving $d' = -\frac{m_1}{m_2}d$

Negative sign shows that particle m_2 should be displaced towards the centre of mass of the system.

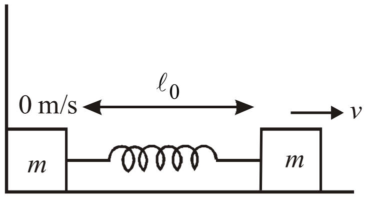
22. (a) We know $m_1r_1 = m_2r_2 \Rightarrow m \times r = \text{constant} \therefore r \propto \frac{1}{m}$

23. (a) Depends on the distribution of mass in the body.

24. (a) $m_1r_1 = m_2r_2 \Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1} \therefore r \propto \frac{1}{m}$

25. (b) $a_{cm} = \frac{m_1a_1+m_2a_2}{m_1+m_2} = \frac{m\times 0+m\times 3g}{m_1+m_2} = \frac{3g}{2}$

26. (b)



By COE, $\frac{1}{2}k\left(\frac{3mg}{k}\right)^2 = \frac{1}{2}mv^2$

$v = \sqrt{\frac{9mg^2}{k}} = 3g\sqrt{\frac{m}{k}}$

$v_{cm} = \frac{m\times 0+mv}{m+m} = \frac{v}{2} = \frac{3g}{2}\sqrt{\frac{m}{k}}$

27. (a) By COE in CM-frame, $\frac{1}{2}\mu v_{ref}^2 = \frac{1}{2}kx^2$

$\frac{1}{2}\frac{m}{2}\left(3g\sqrt{\frac{m}{k}}\right)^2 = \frac{1}{2}kx^2$

$\frac{9}{2}g^2\frac{m^2}{k} = kx^2 \quad ; \quad x = \frac{3mg}{\sqrt{2}k}$

28. (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

29. (a) Initially the electron and proton were at rest so their centre of mass will be at rest. When they move towards each other under mutual attraction then velocity of centre of mass remains unaffected because external force on the system is zero.

30. (d) The centre of mass of a system of particles depends only on the masses of particles and the position of the particles relative to one another. The location of reference frame will not affect the location of centre of mass.